

S-wave meson scattering up to 2 GeV and its spectroscopy

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Abstract. We have performed a thorough study of the meson-meson S-waves with isospin (I) 0 and $1/2$, up to $\sqrt{s} \simeq 2$ GeV. This is the first study that includes 13 channels that have their threshold below that energy. All the resonances below 2 GeV, namely the $f_0(600)$ or σ , $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ and $f_0(1790)$ for $I = 0$, and the $K_0^*(800)$ or κ , $K_0^*(1430)$ and $K_0^*(1950)$ for $I = 1/2$, are generated. We can then extract a clear picture of the spectroscopy, finding that the $f_0(1710)$, together with an important contribution to the $f_0(1500)$, are glueballs. Another pole, which corresponds mainly to the $f_0(1370)$, is a pure octet $I = 0$ state, and does not mix with the glueball.

Keywords: meson-meson scattering, scalar resonances, glueball

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INTRODUCTION

The scalar dynamics is a complicated one due to the large number of resonances and coupled channels that involves. In addition, some of these resonances are very broad, overlap between each other and are very sensitive to the coupled channels involved. Another interesting topic is the study of the nature of these resonances which, in many cases, goes beyond the simple $q\bar{q}$ picture. E.g., one can find in addition dynamically generated resonances, glueballs, etc. All these reasons motivate our study [1], on which we briefly report here, of the $I = 0$ meson-meson S-wave in terms of 13 coupled channels, namely $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$, $\sigma\sigma$, $\rho\rho$, $\omega\omega$, $K^*\bar{K}^*$, $\omega\phi$, $\phi\phi$, $a_1(1260)\pi$ and $\pi^*(1300)\pi$. Simultaneously, we study the S-wave of $K^-\pi^+$ (involving $I = 1/2$ and $3/2$) with the coupled channel scheme, including $K\pi$, $K\eta$ and $K\eta'$.

FORMALISM

To calculate our scattering amplitudes, we use the lowest order $SU(3)$ Chiral Perturbation Theory Lagrangian, \mathcal{L}_2 , and the lowest order interaction chiral Lagrangian of an octet and singlet of 0^{++} resonances, \mathcal{L}_S [2]. The π , K and η form the octet of the lightest pseudoscalar Goldstone bosons. However, when considering higher energy regions, as we do here, one has to take into account additionally the $\eta\eta'$ and $\eta'\eta'$ channels. In the large N_c limit, the η_1 becomes the ninth Goldstone boson. This fact can be used to build chiral Lagrangians based on $U(3)$ symmetry rather than on $SU(3)$, including then the η_1 field. It is well known that the η_1 and η_8 mix to give the physical η and η' mesons, and we take for the mixing angle the value $\sin\theta = -1/3$.

The matrix $\Phi = \sum_{i=1}^8 \phi_i \lambda_i / \sqrt{2} + \eta_1 / \sqrt{3}$ incorporates in a standard way the nonet of the lightest pseudoscalars. We also employ the matrix $U = \exp(i\sqrt{2}\Phi/f)$ and the covariant derivative $D_\mu U = \partial_\mu U - ir_\mu U + iU \ell_\mu$, with f the pion decay constant in the chiral limit fixed to $f_\pi = 92.4$ MeV. The classical external left and right fields, respectively, l_μ and r_μ , are needed to gauge the global chiral symmetry to a local one. We make the identification $v_\mu \equiv (r_\mu + l_\mu)/2 = \lambda W_\mu$, where W_μ is the nonet of the lightest 1^{--} vector resonances (including ρ , K^* , ω and ϕ), and $\lambda = 4.3$ from the $\rho \rightarrow \pi\pi$ width.

The interaction kernels, $N_{i,j}$ (the subscripts i, j represent here the channels), are calculated from the sum of the Lagrangians $\mathcal{L}_2 + \mathcal{L}_S$. \mathcal{L}_2 corresponds to local interactions while \mathcal{L}_S gives rise to the s -channel exchange of $SU(3)$ multiplets of *bare* resonances.¹ We employ the master formula $T = (1 + N \cdot g)^{-1} \cdot N$ [3], where T is a 13×13 matrix that contains the elements $T_{i,j}$ and g is a diagonal matrix with elements $g_i(s)$. The latter are loop functions which represent the two meson s -channel unitarity loop and satisfy a once subtracted dispersion relation [3]. The previous equation embodies the coupled channel interactions driven by the $N_{i,j}$ kernels and unitarity.

In relation to the amplitudes involving the $\sigma\sigma$ channel, we follow a novel method to calculate them without introducing any new free parameter. The σ is a pole due to the $I = 0$ S-wave pion interaction [4]. Then, to calculate the elementary amplitude $A \rightarrow \sigma\sigma$, $N_{A,\sigma\sigma}$, one has to consider first the $A \rightarrow (\pi\pi)_0(\pi\pi)_0$ tree level amplitude from $\mathcal{L}_2 + \mathcal{L}_S$, T_A .² The rescattering of the two pairs of pions is taken into account by multiplying T_A by $1/(D(s_1)D(s_2))$, where s_i is the total center of mass energy squared of the i_{th} pair and $D(s) = 1 + V_2(s)g_{\pi\pi}(s)$ [4], where $V_2(s) = (s - m_\pi^2)/f^2$ calculated from \mathcal{L}_2 . To isolate the $N_{A,\sigma\sigma}$ amplitude, one has to move to the σ pole, s_σ , taking the following limit:

$$\lim_{s_1, s_2 \rightarrow s_\sigma} \frac{T_A}{D_{II}(s_1)D_{II}(s_2)} = N_{A,\sigma\sigma} \frac{g_{\sigma\pi\pi}^2}{(s_1 - s_\sigma)(s_2 - s_\sigma)}, \quad (1)$$

where $g_{\sigma\pi\pi}$ is the σ coupling to $\pi\pi$ and the subscript II indicates that the corresponding function is calculated on the second Riemann sheet, because it is where the σ pole is located. Performing a Laurent expansion of $D_{II}(s)^{-1}$ around s_σ , $D_{II}(s)^{-1} = \alpha_0/(s - s_\sigma) + \dots$, the previous limit reduces to $N_{A,\sigma\sigma} = (\alpha_0/g_{\sigma\pi\pi})^2 T_A$. One can show that $(\alpha_0/g_{\sigma\pi\pi})^2 \simeq f^2$ [1]. Employing $s_i = s_\sigma$ to evaluate $N_{A,\sigma\sigma}$ violates unitarity since then $N_{A,\sigma\sigma}$ would be complex due to the imaginary part of s_σ . To avoid this point, we interpret the large width of the σ as a Lorentzian mass distribution, folding the σ masses ($\sqrt{s_i}$) used to calculate $N_{A,\sigma\sigma}$ and $g_{\sigma\sigma}(s)$ with that distribution [1].

RESULTS AND DATA

From the T -matrix, we can calculate the S -matrix elements, $S_{i,j} = \delta_{i,j} + 2iT_{i,j}\sqrt{q_i q_j}/8\pi\sqrt{s}$, with q_i the centre of mass three-momentum of channel i . The free parameters in our theory are the subtraction constants a_i in the functions $g_i(s)$ and the masses and coupling

¹ Explicit expressions for the simplified situation of just three channels without the η_1 field can be found in ref. [3].

² For definiteness, let us consider $A \neq \sigma\sigma$. The method is easily generalized for that case.

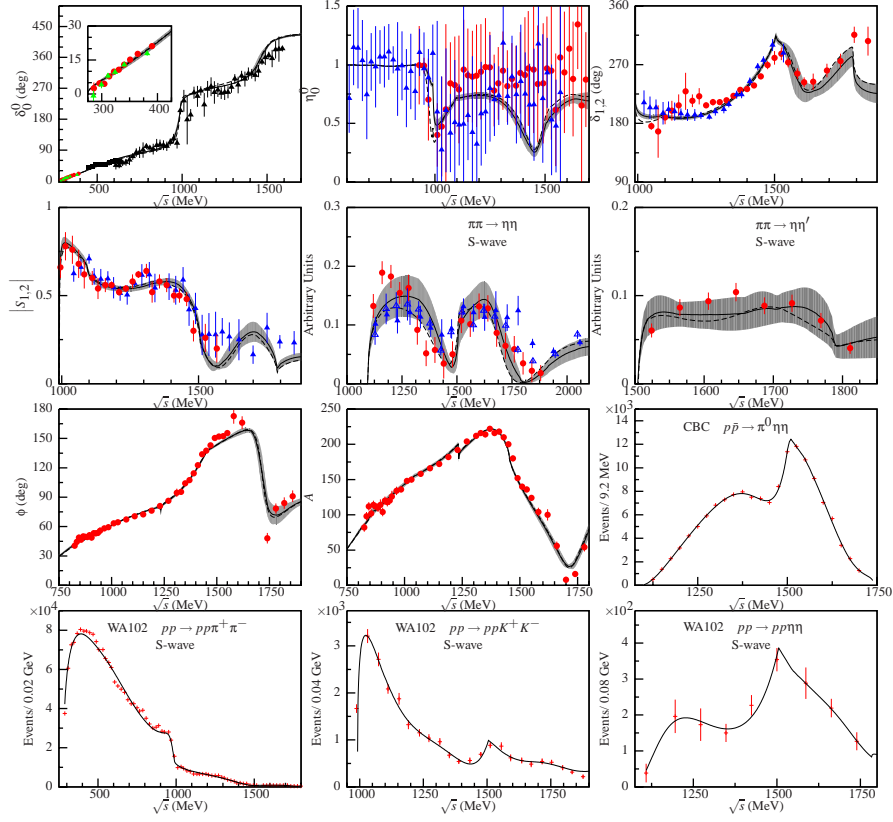


FIGURE 1. From left to right and top to bottom: S -wave $\pi\pi$ phase shift δ_0^0 , elasticity parameter η_0 , phase of the $\pi\pi \rightarrow K\bar{K}$ S -wave $\delta_{1,2}$, its modulus $|S_{12}|$, S -wave event distributions to the $\pi\pi \rightarrow \eta\eta$ and $\eta\eta'$ reactions and the phase (ϕ) and modulus (A) of $K^-\pi^+ \rightarrow K^-\pi^+$. The last four panels correspond to the Crystal Barrel and WA102 Collaborations on $p\bar{p}$ annihilation and pp central production, respectively.

constants involved in \mathcal{L}_S . The number of *free* subtraction constants is reduced because we take $a_{pp} = a_{\omega\omega} = a_{K^*\bar{K}^*} = a_{\omega\phi} = a_{\phi\phi}$, since $SU(3)$ breaking is milder in the vector sector. We can also fix some of the parameters related to the bare resonances required by our fits, two octets and one singlet. Namely, from ref. [5] the first octet is set to $M_8^{(1)} = 1.29$ GeV, $c_d^{(1)} = c_m^{(1)} = 26$ MeV. The mass of the second octet is also fixed, $M_8^{(2)} = 1.90$ GeV from the same reference. So we are left with 3 parameters for the singlet and 2 for the second octet, plus 7 free subtraction constants, totaling 12 free parameters to fit 370 the experimental data of the first six panels in fig.1, from left to right and top to bottom. The data of the last four panels, in the same order, were fitted as a sum of Breit-Wigner's plus a soft background, with the values used for the pole positions and strong couplings to the final states given by the previous fit. In these last data one can clearly observe peaks corresponding to the $f_0(1500)$, $f_0(980)$ and σ . The $f_0(1710)$ is also needed to reproduce the shoulders above 1.5 GeV in several reactions. See ref.[1] for further details. One observes a good reproduction of the data. Compared with previous works, we have fewer free parameters to reproduce more data, and this can be done because we determine the interaction kernels from chiral Lagrangians which allows us to include many more channels and to avoid ad-hoc parametrizations.

TABLE 1. List of the poles found on the different Riemann sheets and couplings of the $f_0(1370)$, f_0^R and $f_0(1710)$. Some branching ratios for the $f_0(1710)$ are also shown.

$I = 0$				$I = 1/2$		
				Pole	$\text{Re}\sqrt{s}$	$\text{Im}\sqrt{s}$
				$f_0(600) = \sigma$	456 ± 6	241 ± 7
				$f_0(980)$	983 ± 4	25 ± 3
				f_0^L	1466 ± 15	158 ± 12
				f_0^R	1602 ± 15	44 ± 15
				$f_0(1710)$	1690 ± 20	110 ± 20
				$f_0(1790)$	1810 ± 15	190 ± 20
Couplings				Branching ratios of $f_0(1710)$		
GeV	$f_0(1370)$	f_0^R	$f_0(1710)$	Value	PDG	
$ g_{\pi^+\pi^-} $	3.59 ± 0.16	1.31 ± 0.22	1.24 ± 0.16	$\frac{\Gamma(K\bar{K})}{\Gamma(\text{total})}$	0.36 ± 0.12	$0.38^{+0.09}_{-0.19}$
$ g_{K^0\bar{K}^0} $	2.23 ± 0.18	2.06 ± 0.17	2.0 ± 0.3	$\frac{\Gamma(\eta\bar{\eta})}{\Gamma(\text{total})}$	0.22 ± 0.12	$0.18^{+0.03}_{-0.13}$
$ g_{\eta\eta} $	1.7 ± 0.3	3.78 ± 0.26	3.3 ± 0.8	$\frac{\Gamma(\pi\pi)}{\Gamma(K\bar{K})}$	0.32 ± 0.14	< 0.11
$ g_{\eta\eta'} $	4.0 ± 0.3	4.99 ± 0.24	5.1 ± 0.8			
$ g_{\eta'\eta'} $	3.7 ± 0.4	8.3 ± 0.6	11.7 ± 1.6			

SPECTROSCOPY

Moving to the complex plane, we find the poles given in table 1. For $I = 1/2$, we have reproduced the resonances found in the PDG. For $I = 0$, we have the σ , $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ and $f_0(1790)$ resonances. The width of $f_0(1710)$ from PDG is 137 ± 8 MeV, which is smaller than 220 ± 40 MeV, as determined from pole position. However, on the real axis, the value of the width corresponding to the half-maximum for the partial waves with $f_0(1710)$ signals is 160 MeV [1], recovering in that way the agreement with PDG. This reduction is due to the opening of several channels along the resonance region. Our determination agrees with the parameters reported by BESII for the $f_0(1790)$. The explanation for $f_0(1370)$ and $f_0(1500)$ is more complicated. The $f_0(1370)$ is mainly given by the pole f_0^L , though the precise shape of the amplitudes on the real axis is sensitive to the f_0^R pole for some channels. This last pole is located on a Riemann sheet which does not influence *directly* the real axis beyond the $\eta\eta'$ threshold, at $\sqrt{s} = 1505$ MeV. This effect typically gives raise to a pronounced signal at the threshold, and that is the reason to have the mass of the $f_0(1500)$ at 1505 ± 6 MeV. From the pole position, one could think that the width is 88 MeV. However, given a Breit-Wigner located at the position of the f_0^R pole, the energy interval below 1.5 GeV at which half the value of the amplitude squared at the maximum (at 1.5 GeV) is reached is $\delta = 1.2\Gamma = 105$ MeV, which is, not by chance, the width of the $f_0(1500)$.

Consider now the couplings given in table 1. The ones of $f_0^L - f_0(1370)$ correspond to the pure $I = 0$ octet member, because they are very close to the bare octet ones [1], calculated from \mathcal{L}_S with $M_8^{(1)}$, $c_d^{(1)}$ and $c_m^{(1)}$ fixed above. This is also the case for the $K_0^*(1430)$ resonance, which is the $I = 1/2$ member of the same octet. So the first octet is a pure one, without mixing with the f_0^R nor $f_0(1710)$. In addition, these couplings imply a large $\pi\pi$ width $\Gamma(f_0(1370) \rightarrow 4\pi)/\Gamma(f_0(1370) \rightarrow 2\pi) = 0.30 \pm 0.12$, in agreement

with the recent determination of ref. [6]. The couplings for $f_0(1710)$ and f_0^R are similar, due to the fact that these poles move into each other in a continuous transition between their respective Riemann sheets. From the couplings of the $f_0(1710)$ we can calculate some branching ratios, given in table 1, together with the values of the PDG, and they are compatible within one sigma. Finally, we obtain that the $f_0(1790)$ has a small $K\bar{K}$ coupling, a major difference with respect to the $f_0(1710)$ as stressed by BESII.

Let us now see that the pattern of the couplings of the f_0^R and the $f_0(1710)$ corresponds to the chiral suppression mechanism of the coupling of a scalar glueball to $\bar{q}q$ [7]. This mechanism predicts that this coupling is proportional to the quark mass, so $\bar{u}u$ or $\bar{d}d$ production is strongly suppressed compared with $\bar{s}s$. With an η - η' mixing angle $\sin\theta = -1/3$, one has that $\eta = -\eta_s/\sqrt{3} + \eta_n\sqrt{2/3}$ and $\eta' = \eta_s\sqrt{2/3} + \eta_n/\sqrt{3}$, where $\eta_s = \bar{s}s$ and $\eta_n = (\bar{u}u + \bar{d}d)/\sqrt{2}$. Denoting by g_{ss} , g_{ns} and g_{nn} the production of $\eta_s\eta_s$, $\eta_n\eta_s$ and $\eta_n\eta_n$, in order, one has $g_{\eta'\eta'} = 2g_{ss}/3 + g_{nn}/3 + 2\sqrt{2}g_{ns}/3$, $g_{\eta\eta'} = -\sqrt{2}g_{ss}/3 + \sqrt{2}g_{nn}/3 + g_{ns}/3$, $g_{\eta\eta} = g_{ss}/3 + 2g_{nn}/3 - 2\sqrt{2}g_{ns}/3$. Taking into account the numerical values given in table 1 for the couplings of the f_0^R we then find $g_{ss} = 11.5 \pm 0.5$, $g_{ns} = -0.2$ and $g_{nn} = -1.4$ GeV, and the suppression is clear. Consider now the $K^0\bar{K}^0$, where $K^0 = \sum_{i=1}^3 \bar{s}_i u^i / \sqrt{3}$, summing over the color indices. The production of a colour singlet $\bar{s}s$ from $\bar{K}^0 K^0$ requires the combination $\bar{s}_i s^j = \delta_i^j \bar{s}s/3 + (\bar{s}_i s^j - \delta_i^j \bar{s}s/3)$, and similarly for $\bar{u}_j u^i$. As only the configuration $\bar{s}s\bar{u}u$ contributes, it picks a factor 1/3. In addition, g_{ss} takes an extra factor 2 compared to $\bar{s}s\bar{u}u$, because the former contains two $\bar{s}s$. One then expects the coupling of $K^0\bar{K}^0$ to have the absolute value $g_{ss}/6$, as it is the case for f_0^R and $f_0(1710)$. Also, quenched lattice QCD [8] agrees with the fact that the coupling of the lightest scalar glueball to pseudoscalar pairs in the $SU(3)$ limit scales as the quark mass, supporting the chiral suppression mechanism, as our results does. This mechanism also implies that the glueball should remain unmixed, which fits with our statement that the $f_0(1710)$ and f_0^R does not mix with f_0^L . In addition, the masses of $f_0(1710)$ and f_0^R agree with the quenched lattice QCD prediction for the lightest scalar glueball, (1.66 ± 0.05) GeV.

In summary, we have presented a detailed coupled channel study of the $I = 0, 1/2$ meson-mesons S -waves up to 2 GeV, including the necessary channels, reproducing the 0^{++} and $1/2^{++}$ resonances below that energy. We have identified the $f_0(1710)$ and f_0^R pole (which is an *important contribution* to the $f_0(1500)$) as glueballs. The pole f_0^L , the main contribution to $f_0(1370)$, turns out to be a pure octet member.

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